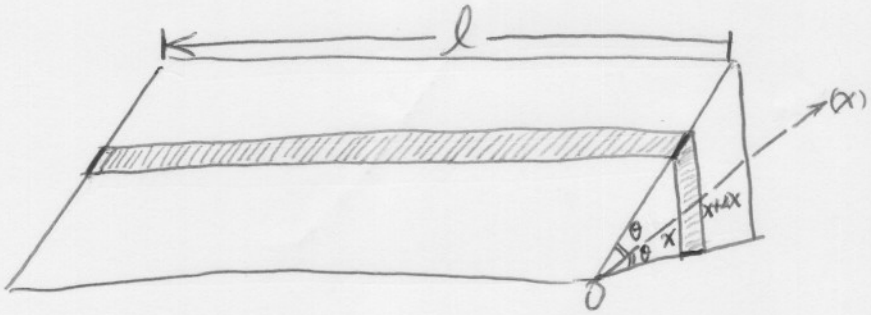
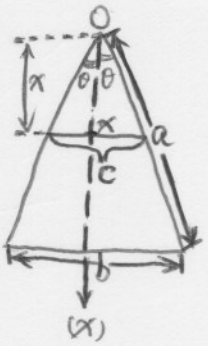




geometric relation:

$$\frac{x}{a\cos\theta} = \frac{c}{b}$$

i.e.: $c = \frac{b}{a\cos\theta}x$

(Note: $\tan\theta = \frac{b}{2a\cos\theta}$)

* Energy entering the control volume:

(1) heat flux through "x" plane

$$\left[-k\frac{dT}{dx}\cdot\Big(\ell\cdot\underbrace{\frac{b}{a\cos\theta}x}_{c}\Big)\right]_x = -k\frac{dT}{dx}\bigg|_x\cdot\Big(\ell\cdot\frac{b}{a\cos\theta}x\Big)$$

* Energy exiting the control volume:

(1) heat flux through "x+Δx" plane

$$\left[-k\frac{dT}{dx}\cdot\Big(\ell\cdot\frac{b}{a\cos\theta}x\Big)\right]_{x+\Delta x} = -k\frac{dT}{dx}\bigg|_{x+\Delta x}\cdot\Big(\ell\cdot\frac{b}{a\cos\theta}\cdot(x+\Delta x)\Big)$$

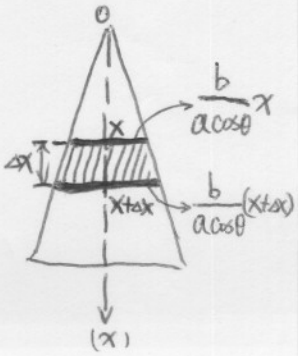
(2) Convective heat transfer through top and bottom planes

$$2\cdot h(T-T_0)\cdot\Big(\ell\cdot\frac{\Delta x}{\cos\theta}\Big)$$

(3) Convective heat transfer through two side planes

$$2\cdot h(T-T_0)\cdot\left[\frac{1}{2}\Big(\frac{b}{a\cos\theta}x + \frac{b}{\cos\theta}(x+\Delta x)\Big)\cdot\Delta x\right]$$

(Can be neglected because of small area)



Energy Balance:

$$\underbrace{\left[-k\frac{dT}{dx}\cdot\left(l\cdot\frac{b}{a\cos\theta}x\right)\right]_x}_{\text{energy in}} = \underbrace{\left[-k\frac{dT}{dx}\cdot\left(l\cdot\frac{b}{a\cos\theta}x\right)\right]_{x+\Delta x} + 2h(T-T_0)\left(l\cdot\frac{\Delta x}{\cos\theta}\right)}_{\text{energy out}}$$

$$\left[k\frac{dT}{dx}\left(l\cdot\frac{b}{a\cos\theta}x\right)\right]_{x+\Delta x} - \left[k\frac{dT}{dx}\left(l\cdot\frac{b}{a\cos\theta}x\right)\right]_x = 2h(T-T_0)\left(l\cdot\frac{\Delta x}{\cos\theta}\right)$$

$$\frac{klb}{a\cos\theta}\left[\left(x\frac{dT}{dx}\right)_{x+\Delta x} - \left(x\frac{dT}{dx}\right)_x\right] = \frac{2lh}{\cos\theta}(T-T_0)\Delta x$$

$$\frac{klb}{a\cos\theta}\left[\frac{d}{dx}\left(x\frac{dT}{dx}\right)\right]\Delta x = \frac{2lh}{\cos\theta}(T-T_0)\Delta x$$

$$\frac{d}{dx}\left(x\frac{dT}{dx}\right) = \frac{2ah}{bk}(T-T_0)$$

define: $T^* \equiv T - T_0$

so: $\frac{d}{dx}\left(x\frac{dT^*}{dx}\right) = \frac{2ah}{bk}T^*$

define: $\lambda^2 \equiv \frac{2ah}{bk}$

so: $\frac{d}{dx}\left(x\frac{dT^*}{dx}\right) - \lambda^2 T^* = 0$

general solution:

$$T^* = A I_0(2\lambda\sqrt{x}) + B K_0(2\lambda\sqrt{x})$$

B.C. $T\Big|_{x=0} = \text{finite}$

so: $B=0$ because $K_0(2\lambda\sqrt{x})\Big|_{x=0} \to \infty$ (for $x=0$)

i.e.: $T^* = A\, I_0(2\lambda\sqrt{x})$

B.C. $T\Big|_{x=a\cos\theta} = T_w$

so: $T^*\Big|_{x=a\cos\theta} = A\, I_0(2\lambda\sqrt{a\cos\theta}) = T_w - T_0$

$$A = \frac{T_w - T_0}{I_0(2\lambda\sqrt{a\cos\theta})}$$

therefore: $$T^* = (T_w - T_0)\frac{I_0(2\lambda\sqrt{x})}{I_0(2\lambda\sqrt{a\cos\theta})}$$

or:

$$T(x) = T_0 + (T_w - T_0)\frac{I_0(2\lambda\sqrt{x})}{I_0(2\lambda\sqrt{a\cos\theta})}$$

with $\lambda \equiv \sqrt{\dfrac{2ah}{bk}}$

or: $\lambda = \sqrt{\dfrac{h}{k\sin\theta}}$